## PART B

1. If R is a relation in the set of integers such that (a,b)∈R iff 3a + 4b =7n for some integers n, prove that R is an equivalence relation

Solution:

(1) S be the set of all positive integers.

For a∈S, 3a+4a =7a, a is an integer

⇒ (a,a) ∈R

R is reflexive

(2) (a,b) ∈R ⇒ 3a+4b =7n for some integer n.

3b+4a =7b-4b+7a-3a =7(a+b-n) where a+b-n is also an integer

⇒ (b,a) ∈R

R is symmetric

(3) (a,b) ∈R ⇒3a+4b =7n

(b,a) ∈R ⇒3b+4a =7m where m, n are integers.

3a+4b+3b+4c =7m+7n

3a+4c =7(n+m-b) where m+n-b is an integer.

⇒ (a,c)∈R

R is transitive

Hence R is an equivalence relation

1. If R is the relation on the set S of positive integers such that (a,b) ∈R if and only if ab is a perfect square, show that R is an equivalence relation

Solution:

(1) S be the set of all positive integers

a.a = a2 is a perfect square

⇒ (a,a) ∈R

R is reflexive

(2) (a,b) ∈R ⇒ ab is a perfect square

⇒ ba is a perfect square

⇒ (b,a) ∈R

R is symmetric

(3) (a,b) ∈R ⇒ab is a perfect square

ab = x2

(b,c) ⇒ bc is a perfect square

bc = y2

ab\*bc = x2y2 ⇒ ab2c = (xy)2 ⇒ ac=(xy/b)2

ie., ac is a perfect square ⇒ (a,c)∈R

R is transitive

Hence R is an equivalence relation

1. Prove that 8n-3n is divisible by 5, n≥1 by Mathematical Induction

Solution: Let P(n): 8n-3n is divisible by 5

* + 1. P(1)= 81-31 = 5 is multiple of 5
    2. Assume that P(k) is true

8k-3k is a multiple of 5 ie., 8k-3k =5r where r is an integer

* + 1. Consider the statement P(k+1)

8k+1-3k+1 = 8k.8-3k.3 = 8.(5r+3k) -3k.3 = 40r +8.3k-3k.3

= 40r-5.3k =5(8r-3k) is a multiple of 5

P(k+1) is true

P(n) is true for all n greater than or equal to 1.

1. If f: R→R g: R→R are defined by f(x) = x2-2, g(x) = x+4, find (fog) and (gof) and check whether these functions are injective, surjective and bijective

Solution:

fog(x) = f[g(x)] = f(x+4) =(x+4)2-2 = x2+8x+14-----------------(1)

gof(x) = g[f(x)] = g(x2-2) = x2+2---------------------------------(2)

Given f: R→R g: R→R

f(x) = x2-2

1. f(1) = 11-2 = -1

f(-1) = (-1)2-2 = -1

i.e., f(x1) = f(x2) does not imply x1 = x2

Hence f is not 1-1 function

(2) Let f: R→R

Let y∈R. Suppose x∈R such that f(x) = y

x2-2 = y

x2 = y+2

x =√y+2

f(√y+2) = (√y+2)2-2=y+2-2 = y

for any y∈R There exist at least one element √y+2∈R such that

f(√y+2)=y

∴ f is on to function

g(x) = x+4

(1) g(x1) = g(x2)

x1+4 = x2+4

x1 = x2

g is 1-1 function

(2) g: R→R

Let y ∈R. Suppose x∈R such that f(x) = y

x = y-4 for any y∈R

There exist at least one element y-4∈R such that

g(y-4) = y

∴ g is on to function

As f is not 1-1 but onto, f is not bijective

As g is 1-1 and onto, g is bijective

1. There are 2500 students in a school. Of these 1700 have taken course in ‘C’, 1000 have taken a course in Pascal, 550 a course in networking, further 750 have taken a course in both C and Pascal, 400 taken course in both Pascal and Networking and 275 have taken course in both Networking and C. If 200 of these students have taken courses in C, Pascal and Networking how many of these 2500 have taken any of these 3 courses C, Pascal and Networking. How many of 2500 have not taken any of these 3 courses C, Pascal and Networking.

Solution:

Let A be the set of students who have taken course in C

Let B be the set of students who have taken course in Pascal

Let C be the set of students who have taken course in networking

Let E be the total students in the school

|A|=1700

|B|=1000

|C|=550

|E|=2500

|A∩B|=750

|B∩C|=400

|A∩C|=275

|A∩B∩C|=200

By the principle of inclusion and exclusion,

|A∪B∪C| = |A|+|B|+|C|-|A∩B|-|A∩C|-|B∩C|+|A∩B∩C|

=1700+1000+550-750-400-275+200

= 2025

∴2025 of 2500 students have taken any course in C, Pascal and Networking

Number of students have not taken any of the 3 course is =2500-2025 =475

1. If f: A→B and g: B→C both 1-1 onto functions, then (gof): A→C. Prove that

1. (gof) is 1-1 onto function

2. (gof)-1 = f-1og-1

Solution

1) Given: f: A→B and g: B→C both 1-1 onto functions

Let x∈A, y∈B, z∈C and y = f(x); z = g(y)

f is 1-1. Therefore f(x1) = f(x2) ⇔ x1 = x2

Also g is 1-1. Therefore g(y1) =g(y2) ⇔ y1 = y2

Let x1, x2∈A

gof(x1) =gof(x2) ⇔ g(f(x1)) =g(f(x2))

⇔ f(x1) = f(x2) since g is 1-1

⇔ x1 = x2 since f is 1-1

(gof)(x1) = (gof)(x2) ⇔ x1= x2

gof: A→C is 1-1 function.

We have to prove that for every z∈C ∃ x∈A such that z = (gof)(x)

g: B→C is onto and z∈C. Therefore ∃ y∈B such that z = g(y)

Since f: A→B is onto and y∈B, ∃ x∈A such that y = f(x)

Thus corresponding to every z∈C ∃ x∈A such that (gof)(x) = g(f(x)) = g(y) = z

### Hence gof is onto function

2) f: A→B g: B→C

gof: A→C (gof)-1: C→A

gof, f, g are one-one onto functions. (gof)-1, f-1, g-1 exist and all are 1-1 onto functions.

Again f-1: B→A

g-1: C→B

f-1o g-1: C→A

Thus both (gof)-1 and f-1og-1 are defined from C→A

y =f(x) and z=g(y) then

(gof)(x) =g(f(x)) =g(y) =z

y =f(x) ⇒x = f-1(y)

z =g(y) ⇒y = g-1(z)

(gof)-1(z) = x (1)

(f-1 o g-1) (z) = f-1( g-1(z))= f-1(y) =x

(f-1 o g-1) (z) = x (2)

from (1) and (2)

(gof)-1(z)= (f-1 o g-1) (z) ∀ z∈C

* (gof)-1 = f-1og-1

1. How many integers between 1 and 300 are
   1. not divisible by any of the integers 3,5,7
   2. divisible by 5 but by neither 3 nor 7

Solution:

Let A be the integers divisible by 3

Let B be the integers divisible by 5

Let C be the integers divisible by 7

|A∩B∩C|=300/3\*5\*7=2

|A|=300/3=100

|B|=300/5=60

|C|=300/7=42

|A∩B|=300/3\*5=20

|B∩C|=300/5\*7=8

|A∩C|=300/3\*7=14

By the principle of inclusion and exclusion,

|A∪B∪C| = |A|+|B|+|C|-|A∩B|-|A∩C|-|B∩C|+|A∩B∩C|

=204-42=162

Not divisible by 3, 5, 7 =|A∪B∪C|c = 300-162 =138

### Divisible by 5 but by neither 3 nor 7 = |B| - |A∪C| =|B|-{|A∩B|∪|B∩C|}

= |B|-{|A∩B|+|B∩C|-|A∩B∩C|}

= 60-20-8+2 =34

1. Prove that (i) (A-C) ∩ (C-B) = Φ and (ii) A-(B∩C) = (A-B)∪ (A-C)

Solution:

(i) (A-C) ∩ (C-B) ={x: x∈A and x∉C and x∈C and x∉B}

={x: x∈A and (x∈C and x∈Cc) and x∈Bc}

={x: (x∈A and x∈Φ) and x∈Bc}

={x: (x∈Φ) and x∈Bc}

={x: (x∈Φ∩Bc}

= Φ

(ii) A-(B∩C) ={x: x∈A and x∉ B∩C}

={x: x∈A and (x∉B or x∉C)}

={x: (x∈A and x∉B) or (x∈A and x∉C}

={x: (x∈ A-B) or (x∈ A-C)}

= (A-B)∪ (A-C)

1. Let R denotes that relation on the set of all ordered pairs of +ive integers by (x,y)R(u,v) iff xv = yu . Show that R is an equivalence relation

Solution:

Let u,v are +ive inegers

(x,y)R(u,v) iff xv = yu

(i) Reflexive

(x,y)R(x,y)

xy = yx for all ordered pairs (x,y) of +ive integers.

∴R is reflexive

(ii)Symmetric

(x,y)R(u,v) ⇒ xv = yu

* yu = xv
* uy = vx

⇒ (u,v) R (x,y)

∴R is symmetric

(iii)Transitive

Let (x,y) (u,v) and (m,n) are ordered pairs of +ive integers

(x,y)R(u,v) and (u,v) R (m,n)

⇒ xv = yu and un = vm

⇒ xvun and yuvm

⇒ xn and ym

⇒ (x,y) R (m,n)

∴R is transitive

Hence R is equivalence relation

1. Prove that the relation “congruence modulo M” in the set of integer is an equivalence relation

Solution:

Relation R is “Congruence Modulo M”

a ≡b (mod M)

a-b is the multiple of M

aRb means a-b is the multiple of M

1. Reflexive

Let a∈Z

a-a = 0 it is a multiple of M and hence (a,a) ∈R

∴R is reflexive

1. Symmetric

Let a,b ∈Z, Let (a,b) ∈R

aRb ⇒ a-b is a multiple of M

⇒ b-a is a multiple of M

⇒ bRa

∴R is symmetric

1. Transitive

Let a,b,c ∈Z, and (a,b), (b,c)∈R

aRb and bRc ⇒ a-b is a multiple of M and b-c is a multiple of M

⇒ a-b + b-c = a-c is a multiple of M

⇒ aRc

∴R is transitive

Hence R is equivalence relation

1. If R is the relation on the set of +ive integers and (a,b) ∈R iff a2+b is even, prove that R is an equivalence relation

Solution:

(a,b) ∈R ⇔ a2+b is even.

This is possible only if a and b are both even integers or a and b are both odd integers.

1. Reflexive

When a is any +ive integer, a2+a = a(a+1) is an even number.

* (a,a) ∈R

∴R is reflexive

1. Symmetric

Case 1: a and b are +ive even integers

(a,b) ∈R⇒ a2+b is even

Also b2+a is even

(b,a) ∈R

Case 2: a and b are +ive odd integers

(a,b) ∈R⇒ a2+b is even

Also b2+a is even

(b,a) ∈R

In both cases (a,b) ∈R⇒(b,a) ∈R

∴R is symmetric

1. Transitive

Case 1:

Let a,b,c are +ive even integers

(a,b) ∈R⇒ a2+b is even

(b,c) ∈R⇒ b2+c is even

Also a2+c is even

(a,c) ∈R

Case 2:

Let a,b,c are +ive odd integers

(a,b) ∈R⇒ a2+b is even

(b,c) ∈R⇒ b2+c is even

Also a2+c is even

(a,c) ∈R

In both cases (a,b) ∈R, (b,c) ∈R⇒(a,c) ∈R

∴R is transitive

Hence R is equivalence relation

1. (i) Given A = {0, ±1, ±2, ±3} and f:A→I defined by f(x) = (3x3+1), find the range of f.

(ii) Given that A = {1, 2, 3, 4} and B ={x, y, z}, how many functions f: A→B are there?

Solution:

(i) f(x) =3x3+1

f(0) =3(0)3+1 =1

f(1) =3(1)3+1 =4 f(-1) =3(-1)3+1= -2

f(2) =3(2)3+1=25 f(-2) =3(-2)3+1= -23

f(3) =3(3)3+1=82 f(-3) =3(-3)3+1= -80

Range = {1, 4, -2, 25, -23, 82, -80}

(ii):

A = {1, 2, 3, 4}

B = {x, y, z}

There are 34 = 81 functions from A→B

In general, if |A| = m and |B| = n, then there are nm functions f: A→B

1. Prove that only 1-1 onto functions can have inverse function.

Proof:

Let f: A→B be a given map. We have to define f-1: B→A

Since f-1 from B→A is a map no element of B can have more than 1 image in A under f-1

∴No element of B can be the image of more than 1 element of A under f.

∴f: A→B must be 1-1.

Again, since f-1: B→A is map, every element of B must have image in A under f-1

∴Every element of B must be the image of some element of A under f. Therefore, f must be onto function. Thus we find that only 1-1 onto map can have inverse function