**PART A**

1. **What is truth table?**

A truth table is a table that displays the relationships between the truth values of sub-propositions and that of compound proposition constructed from them.

The truth table for the disjunction of two propositions p and q

|  |  |  |
| --- | --- | --- |
| **p** | **q** | **pΛq** |
| T | T | T |
| T | F | F |
| F | T | F |
| F | F | F |

1. **Define conjunction?**

When p and q are any two propositions, the proposition “p and q” denoted by p Λ q and called the conjunction of p and q is defined as the compound proposition that is true when both p and q are true and is always false otherwise.

1. **Define disjunction?**

When p and q are any two propositions, the proposition “p or q” denotd by p V q and called the conjunction of p and q is defined as the compound proposition that is false when both p and q are false and is always true otherwise.

1. **What is the order of precedence for logical connectives?**

The orders of precedence for the logical connectives are

1. The negation operator has precedence over all other logical operators.

Thus ~p Λ q means (~p) Λ q, not ~ (p Λ q).

1. The conjunction operator has precedence over the disjunction operator. Thus p Λ q V r means (p Λ q) V r, but not p Λ (q V r).
2. The conditional and bi-conditional operator’s → and ↔ have lower precedence than other operators. Among them, → has precedence over ↔.
3. **Define conditional proposition?**

If p and q are propositions, the compound proposition “if p, then q”, that is denoted by p → q is called a conditional proposition, which is false when p is true and q is false and true otherwise.

1. **Define tautology and contradiction?**

A compound proposition P = P (p1, p2,- - - - - - - pn)where p1, p2, . . . . . , pn are variables (elementary propositions), is called tautology, if it is true for every truth assignment for p1, p2, … …, pn. P is called contradiction, if it is false for every truth assignment for p1, p2, … …., pn

1. **Define duality law?**

The dual of a compound proposition that contains only the logical operators V, Λ and ~ is the proposition obtained by replacing each V by Λ, each Λ by V each T by F and each F by T, where T and F are special variables representing compound propositions that are tautologies and contradictions respectively. The dual of a proposition A is denoted by A\*.

1. **Define Disjunctive Normal Form?**

A compound proposition which consists of a sum of elementary products and which is equivalent to a given proposition is called a disjunctive normal form (DNF).

1. **Define Conjunctive Normal Form?**

A formula which consists of a product of elementary sums and which is equivalent to a given formula is called a conjunctive normal form (CNF).

1. **Define Principal Disjunctive Normal Form (PDNF)?**

A formula consisting of disjunctions of min-terms in the variables only and equivalent to a given formula is known as its principal disjunctive normal form (PDNF) or the sum of products canonical form of the given formula.

1. **Define Principal Conjunctive Normal Form (PCNF)?**

A formula consisting of conjunctions of max-terms in the variables only and equivalent to a given formula is known as its principal conjunctive normal form (PCNF) or the product of sums canonical form of the given formula.

1. **Define Rules of inference?**

Inference theory is concerned with the inferring of a conclusion from certain hypothesis or basic assumptions called premises, by applying certain principles of reasoning called rules of inference. Two basic rules of inference are called Rule P and Rule T.

Rule P: A premise may be introduced at any point in the derivation.

Rule T: A formula S may be introduced in a derivation if S is tautologically

 implied by any one or more of the preceding formulas in the derivation.

1. **Define Universal Quantifier and Existential Quantifier?**

Many mathematical statements assert that a property is true for all values of a variable in a particular domain, called the universe of discourse. Such a statement is expressed using a universal quantification. It is denoted by the notation (x) P(x) or ∀x P(x), which means “For all x, P(x) is true”.

The existential quantification of P(x) is the proposition “There exists at least one x (or an x) such that P(x) is true” and is denoted by the notation ∃x P(x). The symbol ∃ is called the existential quantifier.

1. **Show that  is a tautology without using truth table.**

 Solution: ⇔ 

 

 

 

 

1. **Write the converse and contra positive of “If the file is not damaged and the** processor is fast, then the printer is slow”.

 Solution: If P: The file is damaged, Q: The processor is fast

 And R: The printer is slow

 Then the given statement is 

 The converse of the above statement is 

 The contra positive is 

1. Determine whether the conclusion C:  follows from the premises  and 

 Solution:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| P | Q | H1: PQ | PQ | H2:( PQ) | C :P |
| T | T | T | T | F | F |
| T | F | F | F | T | F |
| F | T | T | F | T | T |
| F | F | T | F | T | T |

Whenever H1 and H2 have the truth value T, C also has the truth value T (in the third and the fourth rows). So C follows from the premises H1­ and H2.

1. Without constructing the truth table, prove that 

 Solution:

 

 

 

 

 

 

1. Obtain the disjunctive normal form for the formula 

Solution:  

 

1. Find PDNF of PV~Q.

 Solution:

PV~Q ≡ P Λ (Q V ~Q) V (~Q) Λ (P V ~P), by complement law

 ≡ (P Λ Q) V (P Λ ~Q) V (~Q Λ P) V (~Q Λ ~P) by distributive law

 ≡ (P Λ Q) V (P Λ ~Q) V (~Q Λ ~P), by commutative and idempotent laws

1. Write the converse and contra positive of “If the file is not damaged and the processor is fast, then the printer is slow”.

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1. Obtain the PDNF of 

 Solution:

|  |  |  |  |
| --- | --- | --- | --- |
| P | Q |  |  |
| T | T | F | T |
| T | F | F | F |
| F | T | T | T |
| F | F | T | T |

The minterms are PQ,  and 

Hence the PDNF is (PQ) ()()