PART B

1. Give the Truth table for ~ (p v (q Λ r) ↔ ((p v q) Λ (p → r)).

 Solution:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| p | Q | r | q Λ r | p v( q Λ r) ≡ a | ~a | pVq | p→r | (pVq) Λ (p → r) ≡ b | ~a ↔ b |
| T | T | T | T | T | F | T | T | T | F |
| T | T | F | F | T | F | T | F | F | T |
| T | F | T | F | T | F | T | T | T | F |
| T | F | F | F | T | F | T | F | F | T |
| F | T | T | T | T | F | T | T | T | F |
| F | T | F | F | F | T | T | T | T | T |
| F | F | T | F | F | T | F | T | F | F |
| F | F | F | F | F | T | F | T | F | F |

1. Show that ~ ((~p Λ q) v (~p Λ ~q)) v (p Λ q) ≡ p by proving the equivalences of the results.

 Solution:

L.H.S ≡ ~ ((~p Λ q) v (~p Λ ~q)) v (p Λ q)

≡ ~ ((~p Λ (q v ~q))) v (p Λ q)

≡ ~ ((~p ΛT)) v (p Λ q)

≡ ~ (~p) v (p Λ q)

≡ p v (p Λ q)

 ≡ p by absorption law.

1. Without constructing the truth tables, find the principal disjunctive normal form for (~p → q) Λ (q ↔ p)

 Solution:

(~p → q) Λ (q ↔ p) ≡ (p v q) Λ ((q Λ p) v (~q Λ ~p))

 ≡ (p v q) Λ ((p Λ q) v ~ (p V q))

 ≡ (p v q) Λ (p Λ q) v ((p v q) Λ ~ (p v q))

 ≡ ((p v q) Λ (p Λ q)) v F

 ≡ ((p Λ (p Λ q)) V ((q Λ (p Λ q))

 ≡ (p Λ q) V (p Λ q)

 ≡ (p Λ q)

1. Without constructing the truth tables, find the principal conjunctive normal form for (p Λ q) V (~p Λ q Λ r).

Solution:

(p Λ q) V (~p Λ q Λ r)

 ≡ ((p Λ q) V ~p) Λ ((p Λ q) V q) Λ ((p Λ q) V r)

 ≡ (p V ~p) Λ (q V ~p) Λ (p V q) Λ (q V q) Λ (p V r) Λ (q V r)

 ≡ T Λ (~p V q) Λ (p V q) Λ q Λ (p V r) Λ (q V r)

 ≡ ((~p V q) V (r Λ ~r)) Λ ((p V q) V (r Λ ~r)) Λ q V (p Λ ~p) Λ (p V r) V

 (q Λ ~q) Λ (q V r) V (p Λ ~p)

 ≡ (~p V q V r) Λ (~p V q V ~r) Λ (p V q V r) Λ (p V q V ~r) Λ (q V p) Λ

 (q V ~p) Λ (p V r V q) Λ (p V r V ~q) Λ (q V r V p) Λ (q V r V ~p)

 ≡ (~p V q V r) Λ (~p V q V ~r) Λ (p V q V r) Λ (p V q V ~r) Λ ((q V p) V

 (r Λ ~r)) Λ ((q V ~p) V (r Λ ~r) (Omitting repetitions)

 ≡ (~p V q V r) Λ (~p V q V ~r) Λ (p V q V r) Λ (p V q V ~r) Λ

 (p V ~q V r) (Deleting repetitions)

1. **State the rules of Inference theory in predicate calculus.**

**Rule US:**

Universal specification is the rule of inference which states that one can conclude that P(a) is true, if ∀x P(x) is true, where a is an arbitrary member of the universe of discourse.

**Rule ES:**

Existential Specification is the rule which allows us to conclude that P(a) is true, if ∃x P(x) is true, where a is an arbitrary member of the universe, but one for which P(a) is true. Usually we will not know what a is, but know that it exists. Since it exists, we may call it a.

**Rule UG:**

Universal Generalization is the rule which stated that ∀x P(x) is true, if P(a) is true, where a is an arbitrary member of the universe of discourse.

**Rule EG:**

Existential Generalization is the rule that is used to conclude that ∃ x P(x) is true when P(a) is true, where a is a particular member of the universe of discourse.

1. **Prove the following famous Socrates argument using inference theory.**

**All men are mortal.**

**Socrates is a man.**

**Therefore Socrates is a mortal.**

Solution:

Let us use the notations

H(x): x is a man

M(x): x is a mortal.

s: Socrates.

With these symbolic notations the problem becomes

∀x (H(x) → M(x)) Λ H(s) => M(s)

The derivation of the proof is as follows

Step No. Statement Reason

1. ∀x (H(x) → M(x)) P
2. H(s) → M(s) US, 2
3. H(s) P
4. M(s) T, 2, 3
5. **Show that b can be derived from the premises a → b, c → b, d → (a** V **c), d, by the indirect method.**

Solution: Let us include ~b as an additional premise and prove a contradiction.

Step No. Statement Reason

1. a → b P
2. c → b P
3. (a V c) → b T, 1, 2
4. d → (a V c) P
5. d → b T, 3, 4
6. d P
7. b T, 5, 6
8. ~b P
9. b Λ ~b T, 7, 8
10. F T, 9
11. **Prove the implication**

**∀x ( P(x) → Q(x)), ∀x (R(x) → ~ Q(x)) => ∀x( R(x) → ~P(x))**

Solution:

Step No. Statement Reason

1. ∀x (P(x) → Q(x)) P
2. P(a) → Q(a) US, 1
3. ∀x (R(x) → ~ Q(x)) P
4. R(a) → ~Q(a) US, 2
5. Q(a) → ~R(a) T, 4
6. P(a) → ~R(a) T, 2, 5
7. R(a) → ~P(a) T, 6
8. ∀x (R(x) → ~P(x)) UG and 7
9. **Show that the following set of premises is inconsistent.**

**If Rama gets his degree, he will go for a job.**

**If he goes for a job, he will get married soon.**

**If he goes for higher study, he will not get married.**

**Rama gets his degree and goes for higher study.**

Solution:

Let the statements be symbolized as follows

p: Rama gets his degree.

q: He will go for a job.

r: He will get married soon.

s: He goes for higher study.

Then we have to prove that

p → q, q → r, s → ~r, p Λ s are inconsistent.

Step No. Statement Reason

1. p → q P
2. q → r P
3. p → r T, 1, 2
4. p Λ s P
5. p T, 4
6. s T, 4
7. s → ~r P
8. ~r T, 6, 7
9. r T 3, 5
10. r Λ ~r T, 8 , 9
11. F T, 10

 Hence the set of given premises is inconsistent.

1. **Show that  P( a , b) follows logically from  and**

 **W(a , b)**

 Solution:

 Step No. Statement Reason

 1.  Rule P

 2.  Rule US, 1

 3.  Rule US, 2

 4.  Rule P

 5. P(a,b) Rule T, 3 and 4

1. **Show that**

**(p → q) Λ (r → s), (q → t) Λ (s → u), ~(t Λ u) and (p → r) => ~p.**

Step No. Statement Reason

1. (p → q) Λ (r → s) P

2. p → q T, 1

3. r → s T, 1

4. (q → t) Λ (s → u) P

5. q → t T, 4

6. s → u T, 4

7. p → t T, 2, 5

8. r → u T, 3, 6

9. p → r P

10. p → u T, 8, 9

11. ~t → ~p T, 7

12. ~u → ~p T, 10

13. (~t V ~u) → ~p T, 11, 12

 14. ~( t Λ u) → ~p T, 13

15. ~(t Λ u) P

16. ~p T, 14, 15

**12Show that (a → b) Λ (a → c), ~(b Λ c), (d V a) => d.**

Step No. Statement Reason

1. (a → b) Λ (a → c) P
2. a → b T, 1
3. a → c T, 1
4. ~b → ~a T, 2
5. ~c → ~a T, 3
6. ( ~b V ~c ) → ~a T, 4, 5
7. ~( b Λ c) → ~a T
8. ~ (b Λ c) P
9. ~ a T, 7, 8
10. d V a P
11. (d V a) Λ ~a T, 9, 10
12. (d Λ ~a) V (a Λ ~a) T, 11
13. (d Λ ~a) V F T, 12
14. d Λ ~a T, 13
15. d T, 14
16. **Prove that the premises a → (b → c), d → (b Λ ~c) and (a Λ d) are inconsistent.**

Step No. Statement Reason

1. a Λ d P
2. a T, 1
3. d T, 1
4. a → ( b → c) P
5. ( b → c) T, 2, 4
6. ~b v c T, 5
7. d → (b Λ ~c) P
8. ~(b Λ ~c) → ~d T, 7
9. ~b V c → ~d T, 8
10. ~d T, 6, 9
11. d Λ ~d T, 3, 10
12. F T, 11
13. **Show that the premises “one student in this class knows how to write programs in JAVA” and “Everyone who knows how to write programs in JAVA can get a high paying job” imply the conclusion “Someone in this class can get a high-paying job”.**

Let C(x) represent “x is in this class”

J(x) represent “x knows JAVA programming” and

H(x) represent “x can get a high paying job”.

Then the given premises are ∃x (C(x) Λ J(x)) and ∀x (J(x) → H(x)). The conclusion is ∃x (C(x) Λ H(x)).

Step No Statement Reason

1. ∃x (C(x) Λ J(x)) P
2. C(a) Λ J(a) ES, 1
3. C(a) T, 2
4. J(a) T, 2
5. ∀x (J(x) → H(x)) P
6. J(a) → H(a) US, 5
7. H(a) T, 4, 6
8. C(a) Λ H(a) T, 3 , 7
9. ∃x (C(x) Λ H(x)) EG, 8
10. **Show that the conclusion ∀x (P(x) → ~Q(x)) follows from the premises.**

**∃x (P(x) Λ Q(X)) → ∀y (R(y) → S(y)) and ∃y (R(y) Λ ~S(y))**

Step No Statement Reason

1. ∃y (R(y) Λ ~S(y)) P
2. R(a) Λ ~S(a) ES , 1
3. ~(R(a) → S(a)) T, 2
4. ∃y (~(R(y) → S(y)) EG, 3
5. ~∀y(R(y) → S(y)) T, 4
6. ∃x (P(x) Λ Q(x)) → ∀y (R(y) → S(y)) P
7. ~∃x(P(x) Λ Q(x)) T, 5, 6
8. ∀x ~(P(x) Λ Q(x)) T, 7
9. ~ (P(b) Λ Q(b)) US, 8
10. ~P(b) V ~Q(b) T, 9
11. P(b) → ~Q(b) T, 10
12. ∀x (P(x) → ~Q(x)) UG, 11
13. **Prove that**

 **∃x P(x) → ∀x ((P(x) V Q(x)) → R(x)), ∃x P(x), ∃x Q(x) =>**

 **∃x ∃y (R(x) Λ R(y))**

Step No. Statement Reason

1. ∃x P(x) P
2. P(a) ES, 1
3. ∃x Q(x) P
4. Q(b) ES, 3
5. ∃x P(x) → ∀x ((P(x) V Q(x)) → R(x)) P
6. P(a) → ((P(b) V Q(b))→R(b)) ES, US, 5
7. (P(b) V Q(b)) → R(b) T, 2, 6
8. P(b) V Q(b) T, 4
9. R(b) T, 7, 8
10. ∃x R(x) EG, 9
11. R(a) ES, 10
12. R(a) Λ R(b) T, 9, 11
13. ∃y (R(a) Λ R(y)) EG, 12
14. ∃x ∃y (R(x) Λ R(y)) EG, 13
15. **Use the indirect method to prove that the conclusion ∃z Q(z) follows from the premises ∀x (P(x) → Q(x)) and ∃y P(y).**

Let us assume the additional premise ~(∃z Q(z)) and prove a contradiction.

Step No. Statement Reason

1. ∃y P(y) P
2. P(a) ES, 1
3. ~(∃z Q(z)) P
4. ∀z (~Q(z)) T, 3
5. ~ Q(a) US, 4
6. P(a) Λ ~Q(a) T, 2, 5
7. ~(~P(a) V Q(a)) T, 6
8. ~(P(a) → Q(a)) T, 7
9. ∀x (P(x) → Q(x)) P
10. P(a) → Q(a) US, 9
11. (P(a) → Q(a)) Λ ~(P(a) → Q(a)) T, 8, 10
12. F T, 11